2021

AUSTRALIAN MATHEMATICS COMPETITION

AMC.

AUSTRALIAN MATHS TRUST

Intermediate Years 9–10

(AUSTRALIAN SCHOOL YEARS)

DATE

4–6 August

TIME ALLOWED

75 minutes

Instructions and Information

General

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

The answer sheet

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder

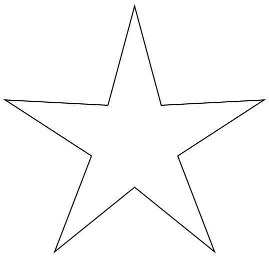
You may sit this competition once, in one division only, or risk no score.

Intermediate Division

Questions 1 to 10, 3 marks each

1. Each edge of this star is 2 cm long.
What is its perimeter?

(A) 5 cm (B) 10 cm (C) 15 cm (D) 20 cm (E) 25 cm



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2. The value of $2000 - 200 + 20 - 2$ is

(A) 1778 (B) 1782 (C) 1818 (D) 1822 (E) 1888

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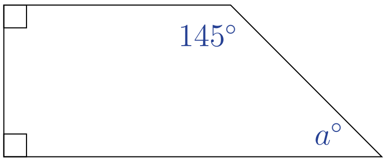
3. What is the value of a in the diagram?

(A) 35 (B) 45 (C) 55 (D) 65 (E) 75



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4. What is 50% more than $\frac{1}{2}$?

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$ (E) 50.5

5.

$$\frac{1+3+5+7+9}{2+4+6+8+10} =$$

(A) $\frac{1}{2}$ (B) $\frac{5}{6}$ (C) $\frac{11}{12}$ (D) $\frac{9}{10}$ (E) $\frac{63}{256}$

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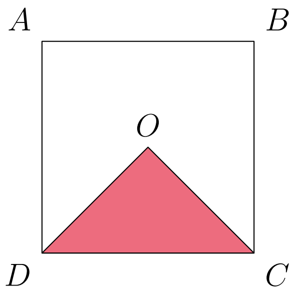
6. Square $ABCD$ has centre O.
The shaded area is 16 square units.
What is the length of the side of the square?

(A) 4 (B) 8 (C) 16 (D) 32 (E) 64



7. On the number line, which number is halfway between 10^2 and 10^4?

(A) 500 (B) 550 (C) 1010 (D) 2021 (E) 5050

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8. To feed a horse, Kim mixes three bags of oats with one bag containing 20% lucerne and 80% oats. If all the bags have the same volume, what percentage of the combined feed mixture is lucerne?

(A) 3 (B) 5 (C) 6 (D) 20 (E) 60

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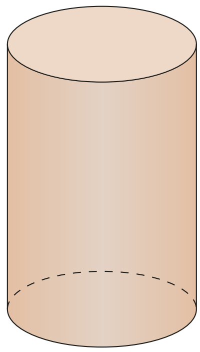
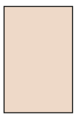
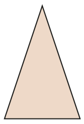
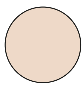
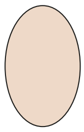
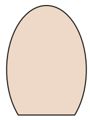
9. I have a solid block of wood in the shape of a cylinder. The top and bottom faces meet the curved side at right angles. Suppose that I slice the cylinder along a plane to create two smaller blocks of wood.

Which of the following could **not** be the shape of the resulting faces created by the slice?



(A)  (B)  (C)  (D)  (E) 

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10. Diya timed herself cycling laps around her suburb. After five laps, her stopwatch indicated a time of 18 minutes and 15 seconds.

What was Diya's average time per lap?

(A) 3 minutes and 3 seconds (B) 3 minutes and 15 seconds
(C) 3 minutes and 27 seconds (D) 3 minutes and 39 seconds
(E) 3 minutes and 51 seconds

Questions 11 to 20, 4 marks each

11. I have four consecutive odd numbers. The largest is one less than twice the smallest. Which of the following is the largest of the four numbers?

(A) 9 (B) 11 (C) 13 (D) 15 (E) 21

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12. On a compact disc, uncompressed music data is stored as 44 100 samples for each second of music, where each sample requires 4 bytes of data. Which of the following is closest to the number of bytes required to store 5 minutes of music on the disc?

(A) 1 million (B) 5 million (C) 10 million (D) 50 million (E) 100 million

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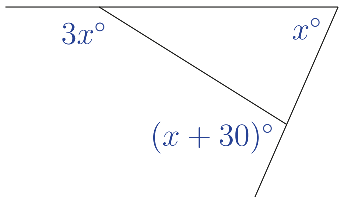
13. In the figure, the value of x is



(A) 30 (B) 40 (C) 50 (D) 60 (E) 70

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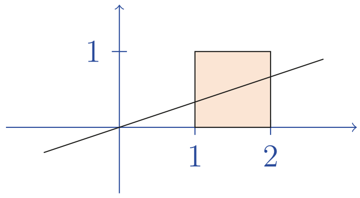
14. What is the equation of the line passing through $(0, 0)$ that bisects the square in the diagram?



(A) $y = \frac{x}{3}$ (B) $y = \frac{x}{2}$ (C) $y = \frac{x}{4}$ (D) $y = 2x$ (E) $y = 3x$

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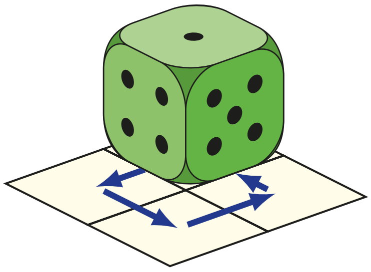
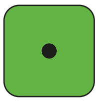
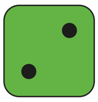
15. A standard dice numbered 1 to 6 with opposite sides adding to 7 is placed on a 2 by 2 square as shown.

The dice is rolled over one edge onto each of the four base squares in turn and then back on to the original square, as indicated by the arrows.

Which side of the dice is now facing upwards?



(A)  (B)  (C)  (D)  (E) 

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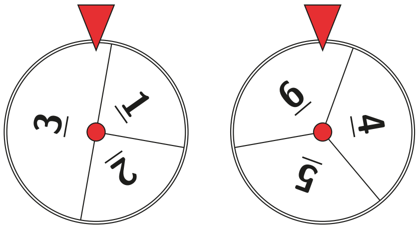
16. The two spinners shown are spun and the numbers that the arrows point to when they stop are recorded.

For example, the numbers here are 3 and 6.

What is the probability that the sum of the two numbers is even?



(A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$ (E) $\frac{5}{12}$

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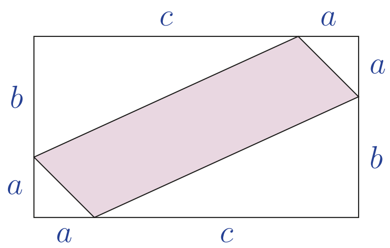
17. The area of the shaded region is given by



(A) $ab + ac$ (B) $a\sqrt{b^2 + c^2}$
(C) $bc + a^2 - ab - ac$ (D) $ab + ac - bc$
(E) $ab + ac - a^2$

18. If k and n are positive integers, and $\sqrt{10n+k}=k$, then the smallest possible value for k is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 10

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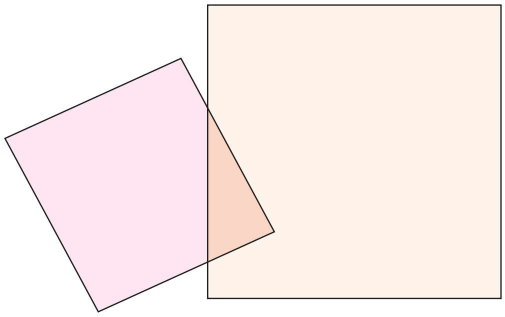
19. Two squares are drawn as shown.

The smaller square covers $\frac{1}{8}$ of the larger square and the larger square covers $\frac{2}{9}$ of the smaller square.

What is the ratio of the side length of the larger square to the side length of the smaller square?



(A) 3:2 (B) 7:3 (C) 7:4 (D) 5:3 (E) 4:3

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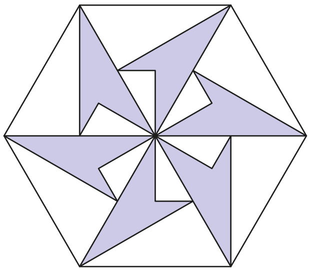
20. Six identical darts fit inside a regular hexagon as shown. Each dart has three interior angles of 30°, and one of 270°. What fraction of the large hexagon is shaded?

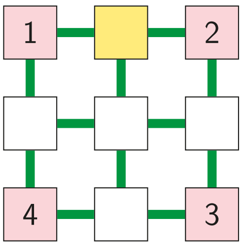


(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{3}{8}$

Questions 21 to 25, 5 marks each

21. We want to place numbers into each of the blank squares in this diagram so that each of the numbers we place is the average of those in the squares directly connected to it.

What number should we put in the middle square of the top row?



(A) $\frac{5}{3}$ (B) $\frac{3}{2}$ (C) $\frac{10}{9}$ (D) $\frac{11}{9}$ (E) $\frac{11}{6}$

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22. To set the timer on his microwave oven, Rick enters the digits of the hours, minutes and seconds in order from left to right. For example, entering '12345' sets the timer to 1 hour 23 minutes 45 seconds, while entering '408' sets it to 4 minutes 8 seconds. One day, Rick accidentally missed the last digit and the timer finished 4 minutes and 42 seconds earlier than he was expecting. What was the missing digit?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

23. I build a large cube from unit cubes. Then I completely paint a number of faces of the large cube. When I dismantle the large cube, I find that I have 288 unit cubes without any paint on them. How many faces of the large cube were painted?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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24. The product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{15^2}\right)$ is equal to

(A) $\frac{7}{13}$ (B) $\frac{8}{15}$ (C) $\frac{9}{16}$ (D) $\frac{10}{21}$ (E) $\frac{13}{24}$

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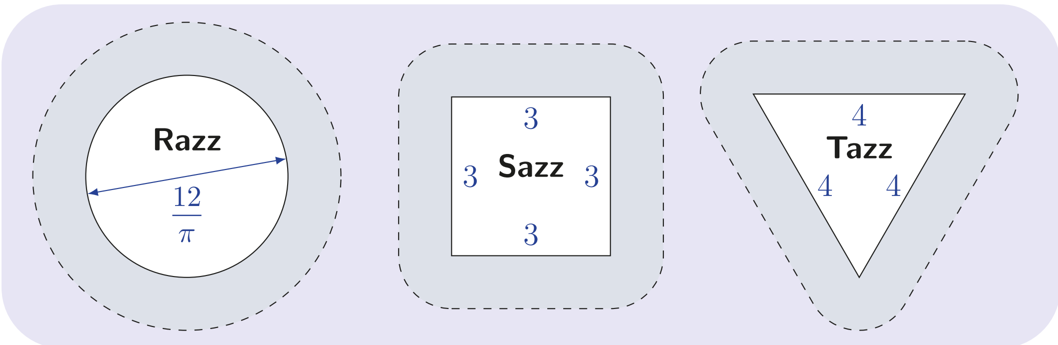
25. Three artificial islands Razz, Sazz and Tazz were constructed in a shallow sea, each with a coastline of 12 km.



Around each island is a fishing zone, consisting of all points in the sea within 1 km of the island. Which islands have a fishing zone of the largest area?

(A) Razz only (B) Sazz only (C) Tazz only
(D) Sazz and Tazz (E) All three have the same area

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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26. In Australian Rules football, a team scores six points for a 'goal' and one point for a 'behind'. During a game, Vladislav likes to record his team's score with a sequence of sixes and ones. There are exactly three distinct sequences which give a final score of 7 points, namely 6,1 and 1,6 and 1,1,1,1,1,1,1.
How many different sequences give a final score of 20 points?

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27. What is the smallest natural number n such that the number

$$N = 100000 \times 100002 \times 100006 \times 100008 + n$$

is a perfect square?

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28. I have a large supply of matchsticks in four colours: red, yellow, blue and green. I use them to make squares where each side is one matchstick long.

I count two squares as the same if one can be rotated and/or reflected to match the shape and colour of the other.

How many different squares can be created?

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29. Bluey divides the number 499 by each of the numbers $1, 2, 3, \ldots, 499$ and records the remainders in order. So her sequence begins:

$$0, 1, 1, 3, 4, 1, \ldots$$

Let M be the sum of these 499 remainders.

Jean-Luc divides the number 500 by each of the numbers $1, 2, 3, \ldots, 500$ and records the remainders in order. So his sequence begins:

$$0, 0, 2, 0, 0, 2, \ldots$$

Let N be the sum of these 500 remainders.

What is the difference between the numbers M and N?

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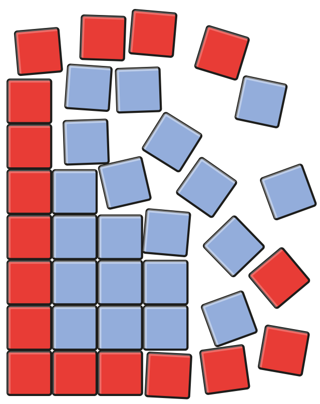
30. Tyler has a large number of square tiles, all the same size. He has four times as many blue tiles as red tiles. He builds a large rectangle using all the tiles, with the red tiles forming a boundary 1 tile wide around the blue tiles.



He then breaks up this rectangle and uses the tiles to make two smaller rectangles. Like the large rectangle, each of the smaller rectangles has four times as many blue tiles as red tiles, and the red tiles form a boundary 1 tile wide around the blue tiles.

How many blue tiles does Tyler have?

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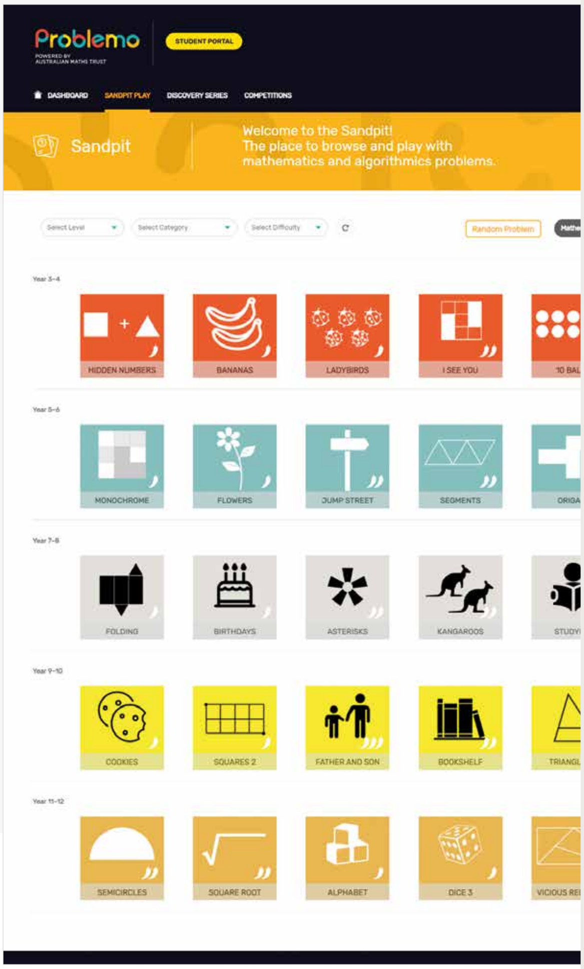

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